



Rossmoyne Senior High School

Semester One Examination, 2020

Question/Answer booklet

MATHEMATICS APPLICATIONS UNIT 3

Section One:
Calculator-free

SOLUTIONS

WA student number: In figures

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In words

Your name

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional
answer booklets used
(if applicable):

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Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
Total					100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free

35% (52 Marks)

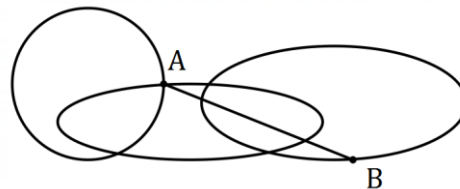
This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1

(6 marks)

Consider the following graph G .



- (a) Draw G in the plane, to clearly show that it is planar.

(2 marks)

Solution
Specific behaviours
<ul style="list-style-type: none"> ✓ 4 edges, 2 labelled vertices ✓ correctly drawn in the plane

- (b) State the degree of each vertex and the degree sum for G .

(2 marks)

Solution
$d_A = 5, \quad d_B = 3$
$d_A + d_B = 8$
Specific behaviours
<ul style="list-style-type: none"> ✓ correct degrees ✓ correct degree sum

- (c) Explain how to recognise a bridge in a connected graph and state, with justification, whether G contains a bridge.

(2 marks)

Solution
When a bridge is removed from a connected graph, the graph becomes disconnected. Yes, edge AB is a bridge in G .
Specific behaviours
<ul style="list-style-type: none"> ✓ correct explanation ✓ identifies edge that is bridge

Question 2

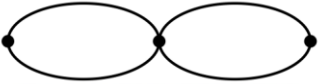
(6 marks)

(a) Connected planar graph G_1 has 3 vertices and 4 edges.

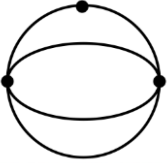


(i) Use Euler's formula to determine the number of faces in G_1 . (2 marks)

Solution
$3 + f - 4 = 2$ $f = 3$
Specific behaviours
<ul style="list-style-type: none"> ✓ correct use of Euler's formula ✓ correct number of faces

(ii) Sketch a possible graph G_1 . (2 marks)

Solution
 <p style="text-align: right;">(Example only - others exist)</p>
Specific behaviours
<ul style="list-style-type: none"> ✓ correct number of edges and vertices ✓ connected, with correct number of faces

(b) Graph G_2 has 3 vertices and is Eulerian. The length of the Euler cycle is 5. Sketch a possible graph G_2 . (2 marks)

Solution
 <p style="text-align: right;">(Example only - others exist)</p>
Specific behaviours
<ul style="list-style-type: none"> ✓ connected with 5 edges and 3 vertices ✓ all vertices with even degree
Markers note
<p><i>Penalise 1 mark overall if vertices not clearly identified.</i></p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  <p>Clearly identified</p> </div> <div style="text-align: center;">  <p>Not clearly identified</p> </div> </div>

Question 3

(6 marks)

A recursive rule for a sequence is $T_{n+1} = T_n + 2.5$, $T_3 = 10.5$.

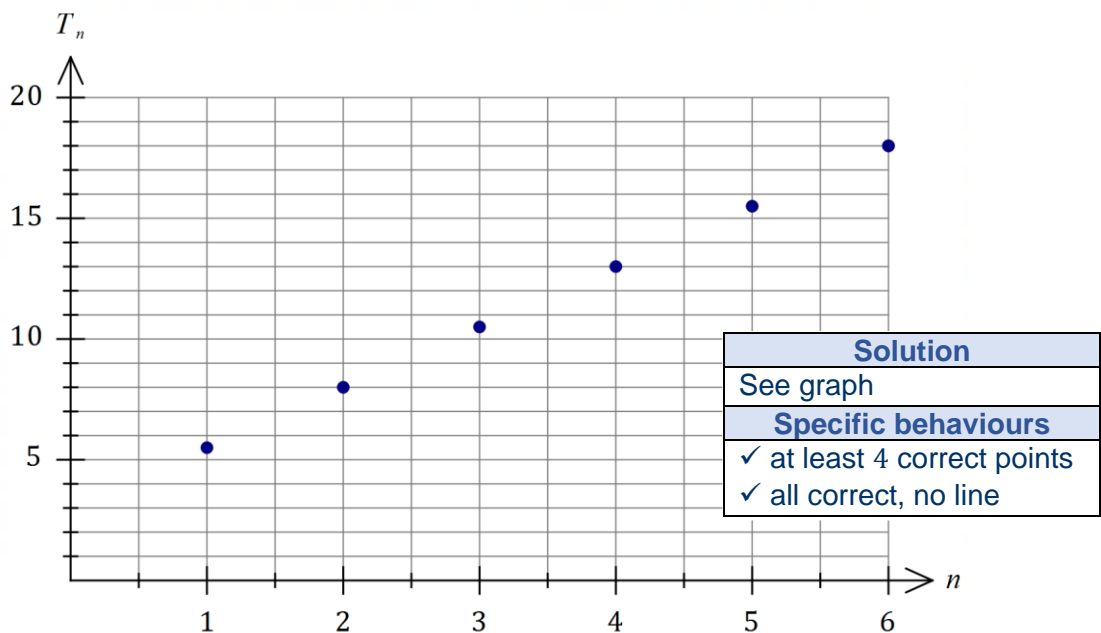
- (a) Briefly explain which feature of the recursive rule indicates that the sequence is arithmetic. (1 mark)

Solution
There is a constant difference of 2.5 between consecutive terms.
Specific behaviours
✓ indicates the constant difference

- (b) Determine T_5 and T_1 . (2 marks)

Solution
$T_5 = 10.5 + 2.5 + 2.5 = 15.5$
$T_1 = 10.5 - 2.5 - 2.5 = 5.5$
Specific behaviours
✓ value of T_5 ✓ value of T_1

- (c) Graph the first six terms of the sequence on the axes below. (2 marks)



- (d) What feature of the graph indicates that the sequence is arithmetic? (1 mark)

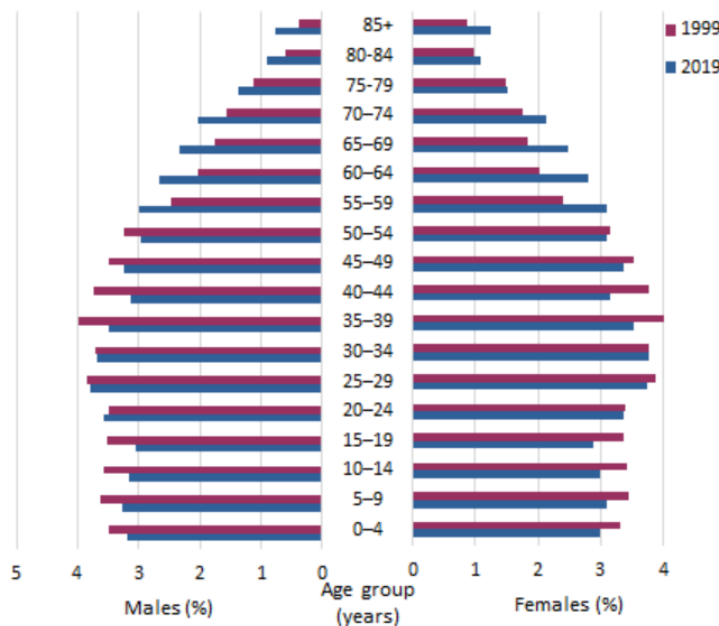
Solution
The points are linear / lie in a straight line.
Specific behaviours
✓ indicates linear nature

Question 4

(7 marks)

Like most developed countries, Australia's population is ageing as a result of sustained low fertility and increasing life expectancy. This has resulted in proportionally fewer children (under 15 years of age) in the population and a larger proportion of people aged 65 and over.

Population change, Age groups - 1999 to 2019



Solution
1999 10%
2019 9%
Specific behaviours
✓ correct amount
✓ correct percentage

a) Calculate the approximate percentage of females under 15 in:

(2 marks)

i) 1999

ii) 2019

b) In 2019 Australia had a population of 26,000,000. Calculate to the nearest 10 000, how many were aged over 70? (2 marks)

Solution
1% = 260,000
10% = 2,600,000 (1 mark)
11% = 2,860,000 (2 marks)
(award 1 mark if only one gender calculated)
Specific behaviours
✓ Calculation is between the boundaries
✓ To the nearest 10 000

c) From the graph above, it appears that there is some association between life expectancy and the decade examined. Explain why, quoting appropriate percentages to support your explanation.

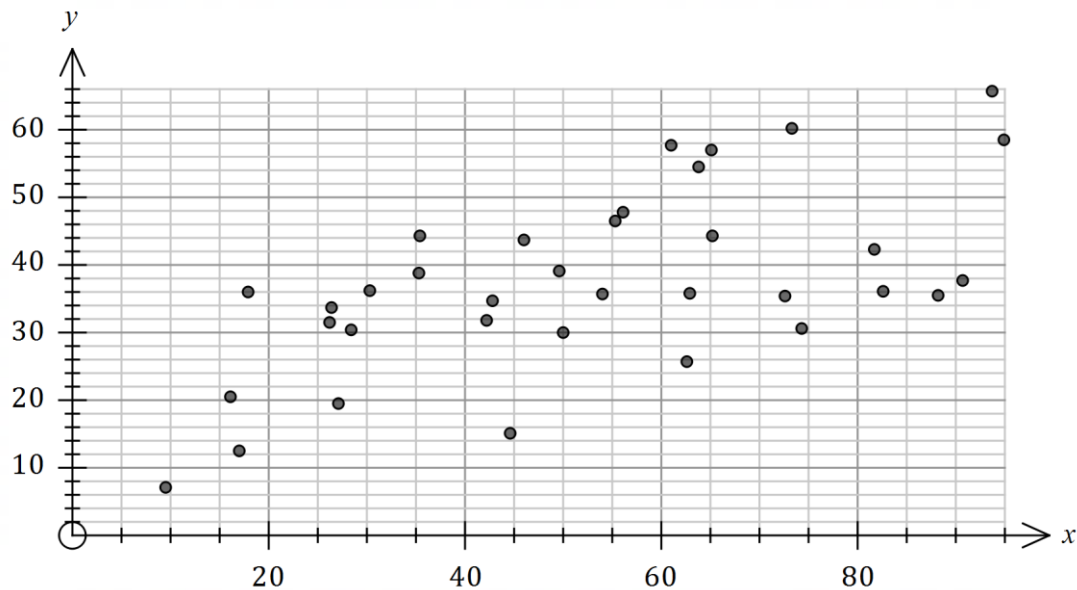
Solution
Examines at least 3 age groups and both sexes
Specific behaviours
✓ considers group under 15
✓ considers group over 65
✓ considers both sexes

(marks)

Question 5

(7 marks)

The scatterplot below shows the number of PC's per 100 people on the x -axis and the GDP/employee, in thousands of dollars, on the y -axis for a selection of countries in 2017.



- (a) Describe the strength and direction of the association between the variables. (2 marks)

Solution
The association is of moderate strength and in a positive direction.
Specific behaviours
<ul style="list-style-type: none"> ✓ strength ✓ direction

- (b) The equation of the least-squares line for the data is $y = 19.1 + 0.35x$. Interpret the intercept and the slope of this line. (3 marks)

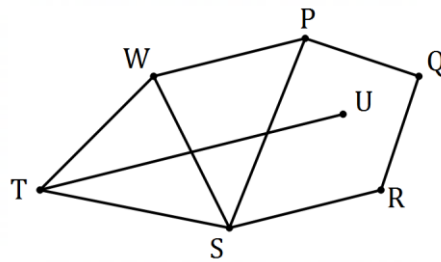
Solution
<p>The intercept means that if a country had no PC's then the GDP/employee is expected to be 19.1 thousand dollars.</p> <p>The slope means that for every extra PC per 100 people, the GDP/employee is expected to rise by 0.35 thousand dollars.</p>
Specific behaviours
<ul style="list-style-type: none"> ✓ interprets intercept ✓ interprets slope ✓ includes units (thousands of dollars) in interpretations

- (c) A newspaper article used the graph to claim that increasing the number of PC's per person in a country caused the GDP/employee to rise. Comment on this claim. (2 marks)

Solution
The claim is unlikely to be true - an observed association does not necessarily mean there is a causal relationship between variables.
Specific behaviours
<ul style="list-style-type: none"> ✓ disputes validity of claim ✓ comments on causation

Question 6

(6 marks)



Graph G is shown. G_1 and G_2 are subgraphs of G , so that each subgraph has 7 vertices but one less edge than G .

- (a) G_1 does not satisfy Euler's formula. State which edge must be removed from G , and show that G_1 does not satisfy Euler's formula. (3 marks)

Solution
Remove edge TU .
Substituting the values $V = 7, F = 4, E = 8$ into the expression $V + F - E$ then $7 + 4 - 8 = 3$ but Euler's formula states this value must equal 2. Hence graph does not satisfy Euler's formula.
Specific behaviours
<ul style="list-style-type: none"> ✓ correct edge to remove ✓ correct values of V, F, E ✓ substitutes and shows formula not satisfied

- (b) G_2 is bipartite. State which edge must be removed from G , and draw G_2 to clearly show the partite sets. (3 marks)

Solution
Remove edge SW .
<i>(Bipartite graphs have no odd length cycles)</i>
Specific behaviours
<ul style="list-style-type: none"> ✓ correct edge to remove ✓ correctly shows partite sets ✓ correctly draws edges

Question 7

(7 marks)

Every day, 25% of the water in a tank is drained for crop irrigation and then the tank is topped up with 50 kL of water. The tank has a maximum capacity of 250 kL.

At the start of Day 1, before water is drained for the crops, the tank contains 40 kL.

- (a) Determine the amount of water in the tank at the start of Day 2. (2 marks)

Solution
$25\% \times 40 = 10$
$40 - 10 + 50 = 80 \text{ kL}$
Specific behaviours
<ul style="list-style-type: none"> ✓ amount drained ✓ correct amount

- (b) Determine a recursive rule for the amount of water, A_n , in the tank at the start of Day n . (2 marks)

Solution
$A_{n+1} = 0.75A_n + 50, \quad A_1 = 40$
Specific behaviours
<ul style="list-style-type: none"> ✓ correct multiplier ✓ correct addition and first term

- (c) Explain why the tank will never empty. (1 mark)

Solution
With this type of recursive rule, the amount in the tank will always increase, tending towards a steady state amount. OR Amount drained is always less than amount added.
Specific behaviours
✓ indicates amount always increasing

- (d) State, with justification, whether the tank will overflow. (2 marks)

Solution
Let x be long-term steady state. $x = 0.75x + 50$ $0.25x = 50$ $x = 200$ Hence tank will not overflow, as the capacity of the tank (250 kL) is more than the long-term steady state (200 kL).
Specific behaviours
<ul style="list-style-type: none"> ✓ indicates correct long-term steady state ✓ states tank will not overflow, with justification

Question 8

(7 marks)

In a set of 5 pages on a website, there are hyperlinks from page 1 to page 5, from page 2 to page 4, from page 3 to page 2, from page 4 to pages 1 and 3, and from page 5 to page 3.

- (a) Construct digraph D to show the above information, where pages are represented by vertices and links by directed edges. (2 marks)

Solution	
Specific behaviours	
✓ vertices all have correct degree ✓ edges all directed correctly	

- (b) Construct an adjacency matrix for D . (2 marks)

Solution																															
	<table style="margin: auto;"> <tr> <td style="padding: 0 10px;">1</td> <td style="padding: 0 10px;">2</td> <td style="padding: 0 10px;">3</td> <td style="padding: 0 10px;">4</td> <td style="padding: 0 10px;">5</td> </tr> <tr> <td style="padding: 5px 10px 5px 10px;">1</td> <td colspan="4" style="padding: 5px 10px 5px 10px;"> $\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$ </td> </tr> <tr> <td style="padding: 5px 10px 5px 10px;">2</td> <td colspan="4"></td> </tr> <tr> <td style="padding: 5px 10px 5px 10px;">3</td> <td colspan="4"></td> </tr> <tr> <td style="padding: 5px 10px 5px 10px;">4</td> <td colspan="4"></td> </tr> <tr> <td style="padding: 5px 10px 5px 10px;">5</td> <td colspan="4"></td> </tr> </table>	1	2	3	4	5	1	$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$				2					3					4					5				
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Specific behaviours																															
✓ at least 4 correct rows ✓ all correct																															

- (c) List, starting at page 4 and in the order visited, vertices in D that form a

- (i) walk of length 2.

Solution
$\{4, 1, 5\}$ or $\{4, 3, 2\}$
Specific behaviours
✓ correct walk

(1 mark)

- (ii) path of length 4.

Solution
$\{4, 1, 5, 3, 2\}$
Specific behaviours
✓ correct trail

(1 mark)

- (iii) cycle of length 3.

Solution
$\{4, 3, 2, 4\}$
Specific behaviours
✓ correct cycle

(1 mark)

Supplementary page

Question number: _____

